Dynamics of Glue-Balls in $N = 1$ SYM Theory

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Abstract

The extension of the Veneziano-Yankielowicz effective Lagrangian with terms including covariant derivatives is discussed. This extension is important to understand glue-ball dynamics of the theory. Though the superpotential remains unchanged, the physical spectrum exhibits completely new properties.

1 Introduction

The low energy effective action of $N = 1$ SYM theory is written in terms of a chiral effective field $S = \varphi + \theta\psi + \theta^2 F$, which may be defined from the local source extension of the SYM action [1, 2, 3, 4]

$$S \propto \frac{\delta}{\delta J} W[J, \bar{J}] \ , \qquad e^{iW[J,\bar{J}]} = \int \mathcal{D}V \ \ e^{i \int d^4x d^2\theta \ (J+\tau_0) \operatorname{Tr} W^\alpha W_\alpha + \text{h.c.}} \ . \tag{1}$$

With appropriate normalization S is equivalent to the anomaly multiplet $\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} = D_\alpha S$. $J(x)$ is the chiral source multiplet, with respect to which a Legendre transformation can be defined [3, 4]. The resulting effective action is formulated in terms of the gluino condensate $\varphi \propto \operatorname{Tr} \lambda\lambda$, the glue-ball operators $F \propto \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} + i \operatorname{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$ and a spinor $\psi \propto (\sigma^{\mu\nu}\lambda)_\alpha F_{\mu\nu}$. An effective Lagrangian in terms of this effective field S has the form [1, 2]

$$\mathcal{L}_{\mathit{eff}} = \int \mathrm{d}^4\theta \ K(S, \bar{S}) - \Big(\int \mathrm{d}^2\theta \ S(\log \frac{S}{\Lambda^3} - 1) + \text{h. c.} \Big) \ . \tag{2}$$

The correct anomaly structure is realized by the superpotential and thus $K(S, \bar{S})$ is invariant under all symmetries. In ref. [1] the explicit ansatz $K = k(\bar{S}S)^{1/3}$ had been made, which leads to chiral symmetry breaking due to $\langle S \rangle = \Lambda^3$, but supersymmetry is not broken as φ and ψ acquire the same mass $m = \Lambda/k$.

2 Glue-balls and constraint Kähler geometry

Though the spectrum found in ref. [1] does not include any glue-balls, such fields do appear in F. However, they drop out in the analysis of [1], as F is treated as an auxiliary field. Indeed, the highest component of a chiral superfield is auxiliary in standard SUSY non-linear σ-models, i.e. there appear no derivatives acting onto this field and moreover its potential is not bounded from below, but from above. In case of the Veneziano-Yankielowicz Lagrangian the part depending on the auxiliary field reads

$$\mathcal{L}_{aux} = k(\bar{\varphi}\varphi)^{-\frac{2}{3}}\bar{F}F + \Big(\frac{1}{3}\varphi^{-\frac{2}{3}}\bar{\varphi}^{-\frac{5}{3}}F\bar{\psi}\bar{\psi} - F\log\frac{\varphi}{\Lambda^3} + \mathrm{h.\,c.}\Big) \ , \tag{3}$$

and the supersymmetric spectrum is obtained, *if and only if* F is eliminated by the algebraic equations of motion that follow from (3). This leads to the unsatisfactory result that glue-balls cannot be introduced in a straightforward way (cf. also [5]) which, in addition, contradicts available lattice-data [6].

However, in the special case of $N = 1$ SYM the elimination of F is not consistent: If F is eliminated from (3), this implies that the theory must be ultra-local in the field F *exactly*, i.e. even corrections to the effective Lagrangian which are not included in (2) are not allowed to change the non-dynamical character of F. If this field would be related to the fundamental auxiliary field, this restriction would be obvious. But in $N = 1$ SYM the situation is different: S is the effective field from a composite operator and F is not at all related to the fundamental auxiliary field D. As a consequence, the restriction of ultra-locality on F leads to an untenable constraint on the *physical* glue-ball operators (for details we refer to [4, 7, 8]).

As shown in ref. [2], the effective Lagrangian of [1] is not the most general expression compatible with all the symmetries, but the constant k may be generalized to a function $k(\frac{S^{1/3}}{\bar{D}^2\bar{S}^{1/2}}, \frac{\bar{S}^{1/3}}{D^2 S^{1/2}})$. This non-holomorphic part automatically produces space-time derivatives onto the field F, which is most easily seen when $K(S,\bar{S})$ is rewritten in terms of two chiral fields [8]:

$$K(S,\bar{S}) \rightarrow K(\Psi_0,\Psi_1;\bar{\Psi}_0,\bar{\Psi}_1) \tag{4}$$

Ψ_0 and Ψ_1 are not independent, but they must obey the constraints

$$\Psi_0 = S^{\frac{1}{3}} = \varphi^{\frac{1}{3}} + \frac{1}{3}\varphi^{-\frac{2}{3}}\theta\psi + \frac{1}{3}\theta^2(\varphi^{-\frac{2}{3}}F + \frac{1}{3}\varphi^{-\frac{5}{3}}\psi\psi) \ , \tag{5}$$

$$\Psi_1 = \bar{D}^2\bar{\Psi}_0 = \frac{1}{3}(\bar{\varphi}^{-\frac{2}{3}}\bar{F} + \frac{1}{3}\bar{\varphi}^{-\frac{5}{3}}\bar{\psi}\bar{\psi}) - \frac{i}{3}\theta\sigma^\mu\partial_\mu(\bar{\varphi}^{-\frac{2}{3}}\bar{\psi}) - \theta^2\Box\bar{\varphi}^{\frac{1}{3}} \ . \tag{6}$$

As F appears as lowest component of $\bar{\Psi}_1$, the Lagrangian includes a kinetic term for that field. In contrast to the situation in [1], this is not inconsistent as the potential in F may include arbitrary powers in that field (instead of a quadratic term only) and can be chosen to be bounded from below (instead of above). This way the field F is promoted to a usual physical field. It has been shown in [7] that there exist consistent models of this type. In [8] these ideas have been applied to $N = 1$ SYM, leading to an effective action of that theory with dynamical glue-balls as part of the low-energy spectrum. Formally, the effective potential looks the same as in the case of Veneziano and Yankielowicz:

$$
\begin{aligned}
V_{\text{eff}} = & -\tilde{g}_{\varphi\bar{\varphi}} F\bar{F} + \frac{1}{2}\tilde{g}_{\varphi\bar{\varphi},\bar{\varphi}} F(\bar{\psi}\bar{\psi}) + \frac{1}{2}\tilde{g}_{\varphi\bar{\varphi},\varphi}\bar{F}(\psi\psi) - \frac{1}{4}\tilde{g}_{\varphi\bar{\varphi},\varphi\bar{\varphi}}(\psi\psi)(\bar{\psi}\bar{\psi}) \\
& + c\big(F\log\frac{\varphi}{\Lambda^3} + \bar{F}\log\frac{\bar{\varphi}}{\bar{\Lambda}^3} - \frac{1}{2\varphi}(\psi\psi) - \frac{1}{2\bar{\varphi}}(\bar{\psi}\bar{\psi})\big)
\end{aligned}
\tag{7}
$$

However, in contrast to [1] the Kähler "metric"[1] is a function of φ *and* F, $\tilde{g}_{\varphi\bar{\varphi}}(\varphi, F; \bar{\varphi}, \bar{F})$. From eq. (7) the consistent vacua can be derived, for explicit expressions we refer to [8]. The most important properties of the Lagrangian (2) with (4) are:

The effective potential is minimized with respect to *all* fields φ, ψ and F. Consequently, the dominant contributions that stabilize the potential must stem from the Kähler part, not from the superpotential: The superpotential is a holomorphic function in its fields and therefore its scalar part must have unstable directions. In the present context there exists no mechanism to transform these instabilities into stable but non-holomorphic terms.

Though the model has the same superpotential as the Lagrangian of ref. [1] its spectrum is completely different: Chiral symmetry breaks by a vacuum expectation value (vev) of $\varphi \propto \Lambda^3$, but this mechanism is more complicated than in [1]. Any stable ground-state must have non-vanishing vev of F. But $\langle F \rangle$ is the order parameter of supersymmetry breaking and thus this symmetry is broken as well[2]. ψ is a massless spinor, the Goldstino.

The supersymmetry breaking scenario is of essentially non-perturbative nature[3]: it is not compatible with perturbative non-renormalization theorems, as the value of V_{eff} in its minimum and the vev of $T^\mu{}_\mu$ are no longer

[1] This quantity is not equivalent to the true Kähler metric of the manifold spanned by Ψ_0 and Ψ_1, cf. [8].

[2] The author of ref. [2] concluded that this model cannot have a stable *supersymmetric* ground-state. This is in agreement with our results, as the model breaks down as $F \to 0$.

[3] The importance of such a breaking mechanism has been pointed out in [4] already, but a concrete description was not yet found therein.

equivalent. In particular, the former can be negative, while the latter is positively semi-definite due to the underlying current-algebra relations. To our knowledge this is the first model, where this type of supersymmetry breaking has found a concrete description (cf. [7, 8] for details).

Any ground state with $\langle \tilde{g}_{\varphi\bar{\varphi}} \rangle \neq 0$ can be equipped with stable dynamics for $p^2 < |\Lambda|^2$. In the construction of concrete kinetic terms it is important to realize that (4) may include expressions with explicit space-time derivatives. Again this is possible as F is not interpreted as an auxiliary field.

In summary, the Lagrangian of ref. [8] is the most general one, which can be formulated in terms of the effective field S. Consistent ground-states can be found together with broken supersymmetry only. It would be interesting to compare these results with a *different* action, which has supersymmetric ground-states. But the "pièce de résistance" for such an action is the fact, that it cannot start from the effective field S.

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